Reversible Occurrence Nets and Causal Reversible Prime Event Structures

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RC 2020@multiple locations

Starting points

Reversible semantics for Place/Transition Nets via enriched unfoldings (Melgratti, Mezzina & Ulidowski)

- each execution of a P/T net is represented as an acyclic net
- common prefixes are identified
- to each transition t of the unfolding, a transition representing the undoing of t is added

Starting points

Reversible semantics for Place/Transition Nets via enriched unfoldings (Melgratti, Mezzina & Ulidowski)

the classic relationship between unfoldings and event structures (too many)

- unfolding = (occurrence net, labelling)
- an occurrence net is equipped naturally with a partial order (acyclic net) and a conflict relation
- basically the same basic ingredients of event structures

Starting points

Reversible semantics for Place/Transition Nets via enriched unfoldings (Melgratti, Mezzina & Ulidowski)

the classic relationship between unfoldings and event structures (too many)

Given these starting points, some questions arise

- how the relationship between nets and event structures can be exploited?
- what would be the net notion associated to reversible event structures?

Plan of the talk

- review the notions of occurrence nets, prime event structures and reversible prime event structures
- introduce reversible occurrence nets
- exploit of the relationship between this new notion and reversible prime event structures, and discuss shortcomings
- provide some thoughts on how to solve the problem in general
- give concluding remarks

Occurrence nets

$C = \langle B, E, F, \mathrm{c} \rangle$

- B are conditions, E are events, c are the initial conditions and F is the flow relation
- acyclic net (the flow relation induces an irreflexive partial order)
- each condition b has at most one predecessor
- each event e has a finite number of predecessors
- conflicts are identified by common branching conditions and inherited along the flow relation (give an irreflexive and symmetric relation)

Occurrence nets

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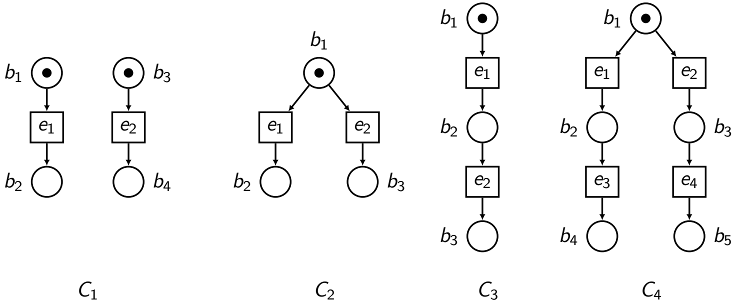

in C_1 e_1 and e_2 are concurrent, in C_2 e_1 and e_2 are in conflict, in C_3 e_1 and e_2 are causally related and in C_4 conflict inheritance is displayed

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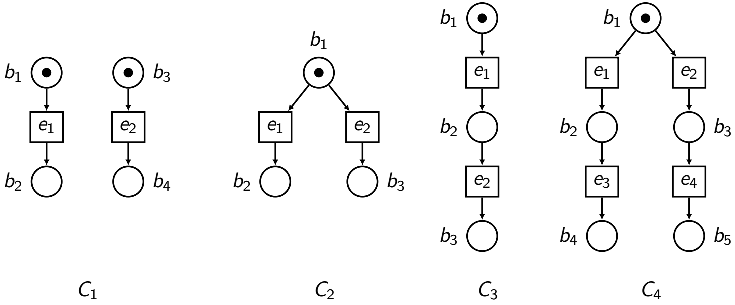

in C_1 e_1 and e_2 are concurrent, in C_2 e_1 and e_2 are in conflict, in C_3 e_1 and e_2 are causally related and in C_4 conflict inheritance is displayed

observe: the dependency between two events arises when a token is in a common place

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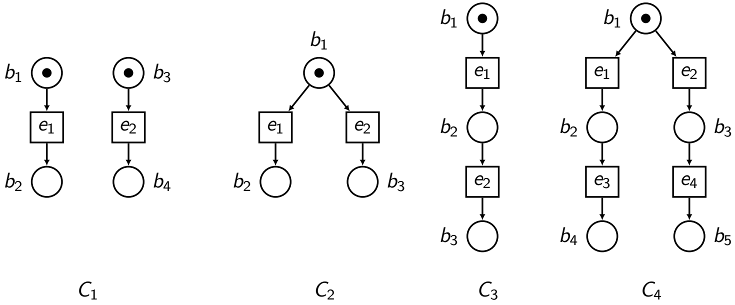

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markings are obtained by taking (the postset of) conflict free and causally closed subsets of events

Prime event structures

$P = (E, <, \#)$

- $<$ is an irreflexive partial order: the causality relation
- each event e has a finite number of predecessors
- $\#$ is an irreflexive and symmetric conflict relation which is inherited along the causality relation

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causality, concurrency and conflict are represented using the two relations

Occurrence nets and prime event structures

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$\mathcal{P}(C) = (E, <, \#)$ is indeed a prime event structure where $<$ is obtained closing transitively F and $\#$ is the conflict relation induced by branching conditions (and inherited along the flow relation)

obvious correspondence between markings and configurations

Occurrence nets and prime event structures

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Given the prime event structure $(E, <, \#)$

- the conditions are made from subsets of conflicting events and some information about the events themselves: the set of conditions certainly contains
  - $(\bot, \{e_1\}), (e_1, \emptyset)$ for each event,
  - $(e_2, \{e_1\})$ for $e_1 < e_2$,
  - $(\bot, \{e_1, e_2\})$ for $e_1 \# e_2$

  and it is saturated
- the flow relation contains $(e, (e, -)$ and $((-, A), e)$ if $e \in A$
- initial marking corresponds to the empty configuration

Occurrence nets and prime event structures

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Given the prime event structure $(E, <, \#)$

- the conditions are made from subsets of conflicting events and some information about the events themselves: the set of conditions certainly contains
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- initial marking corresponds to the empty configuration

$\mathcal{E}(P) = \langle B, E, F, \mathrm{c} \rangle$ is indeed an occurrence net

obvious correspondence between configurations and markings

Reversible prime event structures

$\mathrm{P} = (E, U, <, \#, \prec, \triangleright)$ where

- $U \subseteq E$ are the reversible/undoable events
- reverse events are $\underline{U} = \{\underline{u} \mid u \in U\}$ and are disjoint from E,
- $(E, <, \#)$ is almost a prime event structure,
- $\prec\ \subseteq E \times \underline{U}$ is the *reverse causality*
- $\triangleright\ \subseteq E \times \underline{U}$ is the *prevention* relation

some further requirements:

- to reverse (undo) an event the event itself should have happened and the set of its causes is finite
- reverse causality and prevention do not overlap
- conflicts are inherited along the sustained causation relation $\ll$ (obtained using $<$ and $\triangleright$)

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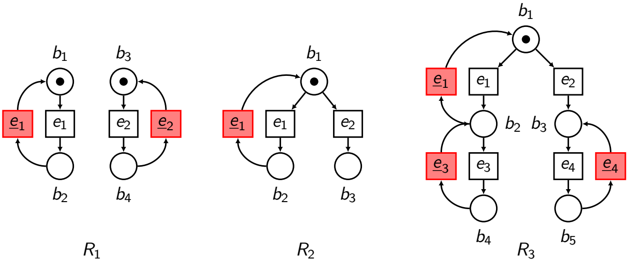
observe: there is an implementation of each reversible event

Reversible occurrence nets

Intuition: take an occurrence net and add some reversing transitions (reversing events)

$R = \langle B, E, U, F, \mathsf{c} \rangle$ with the requirement that the net without the reversing transitions U should be an occurrence net and for each event in U there is a unique counterpart in E



the flow relation for the reversing transitions is deducible from the one for the corresponding transition

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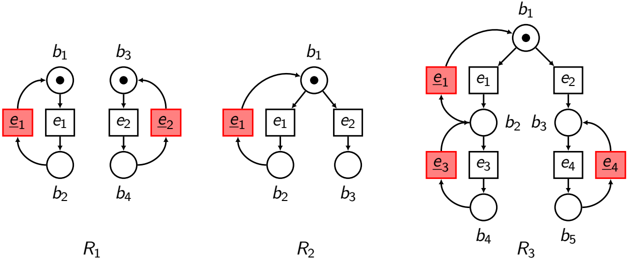

observe: no prevention and reverse causality relations

Intermezzo: Causal reversible prime event structures

Place some further requirements on the relations in $\mathsf{P} = (E, U, <, \#, \prec, \triangleright)$

- P is cause-respecting if two causally related events are also sustained causally related (prevention does not hinder causality)

- P is causal if for any $e \in E$ and $u \in U$
  - $e \prec \underline{u}$ iff $e = u$ (reverse causality is determined by a unique event),
  - $e \triangleright \underline{u}$ iff $u < e$ (the prevention relation is induced by the future of an event)

From reversible occurrence nets to reversible prime event structures

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$\mathcal{C}_r(R) = (E, U, <, \#, \prec, \triangleright)$ where

- $<$ is induced by the flow relation of the occurrence net without revering transitions
- $\#$ is induced by the conflict relation of the occurrence net without revering transitions
- the relation $\prec$ is defined as $e \prec \underline{u}$ where u is the reversing of e
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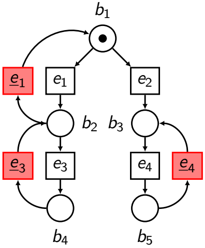
$\mathcal{C}_r(R)$ is a causal reversible prime event structure

prevention and reverse causality have the proper shape because of the occurrence net associated to R

From reversible occurrence nets to reversible prime event structures

from



we get $e_1 \prec \underline{e}_1$, $e_3 \prec \underline{e}_3$, $e_4 \prec \underline{e}_4$ and $e_3 \triangleright \underline{e}_1$, the other relations are the usual ones

About the vice versa

The idea is the same as before: given the reversible prime event structure

$$(E, U, <, \#, \prec, \triangleright)$$

construct an occurrence net from the almost prime event structure $(E, <, \#)$ and then add the reversing transitions corresponding to the events in U

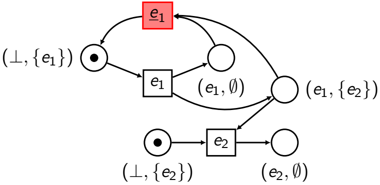
Unfortunately this does not work properly

About the vice versa

From the reversible prime event structure

$$(\{e_1, e_2\}, \{e_1\}, e_1 < e_2, \emptyset, e_1 \prec \underline{e}_1, \emptyset)$$

we get



$\{e_2\}$ is a configuration of the reversible prime event structure but not in the associated reversible occurrence net

About the vice versa

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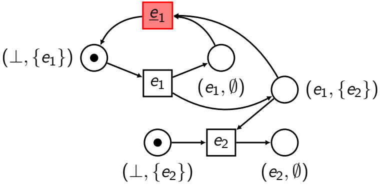

this reversible prime event structure is not a causal one

From causal reversible prime event structures to reversible occurrence nets

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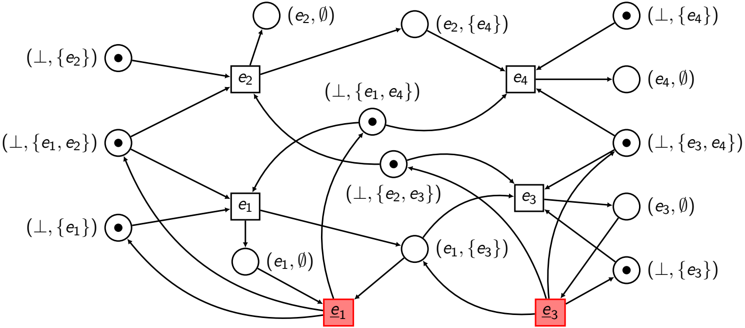
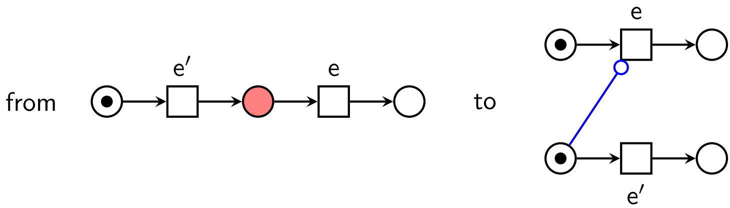
Consider the causal reversible prime event structures with four events $\{e_1, e_2, e_3, e_4\}$, the reversible ones are $\{e_1, e_3\}$ and where the causality is $e_1 < e_3$ and $e_2 < e_4$, conflicts are $e_1 \# e_2, e_1 \# e_4, e_2 \# e_3$ and $e_2 \# e_4$, the reverse causality is $e_1 \prec \underline{e}_1$ and $e_3 \prec \underline{e}_3$ and the prevention relation is $e_3 \triangleright \underline{e}_1$



Rethinking causality in nets

Causality between two transitions is represented via a common place, but



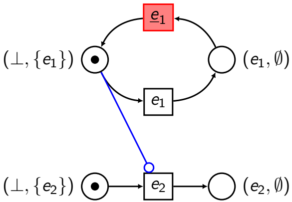
and one may ask if this different view of causality could be of help in solving problems

Rethinking causality in nets

From the reverse prime event structure

$$(\{e_1, e_2\}, \{e_1\}, e_1 < e_2, \emptyset, e_1 \prec \underline{e}_1, \emptyset)$$

we could get something of this kind



now $\{e_2\}$ is a configuration of this net with reversing events

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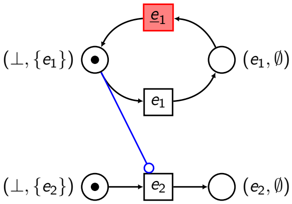

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reverse causality (non mandatory one) could be modelled with read arcs and prevention with inhibitor arcs as causality (but from different conditions!)

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- a straightforward adaptation of the usual notion of occurrence net
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- it fits with the intuition that before undoing an event also its consequences should be undone

Conclusions and future works

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- a straightforward adaptation of the usual notion of occurrence net
- it works perfectly with causal reversible prime event structure
- it fits with the intuition that before undoing an event also its consequences should be undone
- what about a categorical treatment? not only morphisms but also constructions
- how to solve the problem in general: which kind of net corresponds to reversible prime event structures?

Thank you!